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27532

B.C.A. EXAMINATION, May 2019

(Second Semester)

MATHEMATICAL FOUNDATIONS-II

BCA-123

Time : 3 Hours]   [Maximum Marks : 80

Note : Attempt *Five* questions in all, selecting at least *one* question from each Unit. Q. No. 1 is compulsory. All questions carry equal marks.

1. (a) Find the characteristic roots of the matrix : 2½

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

(b) If a matrix has 36 elements, what are the possible orders it can have ? 3

(c) Define rank of a matrix and given one example. 2½

(d) Define order of an elements of a group. 2½

(e) Check whether the sentence "May God Bless You" is a statement or not, give reasons. 3

(f) Identify the quantifiers and write the negation of the statement. All cars are not fast and safe. 2½

Unit I

2. (a) Using principle of mathematical induction, prove that for all $n \in N$:

$$\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots\cdots + \frac{1}{2^n} = 1 - \frac{1}{2^n}$$

(b) Construction the truth table of the following compound statements :

(i) $(p \wedge q) \vee \sim (p \vee q)$

(ii) $\sim p \vee (q \wedge \sim r)$

3. (a) Show that conditions connective is neither commutative nor associative.

 (b) Prove that $p \wedge (\sim p \vee \sim q)$ is neither a tautology nor a contradiction.

Unit II

4. (a) Show that the set of all non-zero rational numbers forms an abelian group under the operation of multiplication of rational numbers. https://www.crsuonline.com

 (b) Prove that $(\{0, 1, 2, 3, 4\}, +_5, \times_5)$ is a field.

5. (a) Show that the intersection of any *two* left ideals of a ring is a left ideal of the ring.

 (b) Prove that the set $\{0, 1, 2, 3, 4, 5\}$ with addition modulo 6 and multiplication modulo 6 as compositions is a ring.

Unit III

6. (a) Express $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$ as a product of elementary matrices.

 (b) If $a \neq b$ and x, y and z are not all zero and if :

$$ax + by + cz = 0$$
$$bx + cy + az = 0$$
$$cx + ay + bz = 0$$

prove that $a : b : c = 1 : 1 : 1$

7. (a) Find the rank of the matrix :

$$A = \begin{bmatrix} 9 & 0 & 2 & 3 \\ 0 & 1 & 5 & 6 \\ 4 & 5 & 3 & 0 \end{bmatrix}$$

by reducing it to normal form.

 (b) Prove that every square matrix A can be expressed in one and only one way as $\phi + i\theta$, where ϕ and θ are Hermitian matrices.

Unit IV

8. (a) Prove that the absolute value of each characteristic root of unitary matrix is unity.

(b) Find the eigen values and eigen vectors of the matrix :

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

9. (a) Diagonalize the matrix :

$$\begin{bmatrix} 6 & 0 & 0 \\ 0 & 7 & -4 \\ 9 & 1 & 3 \end{bmatrix}$$

(b) If λ is an eigen value of a square matrix A, then prove that $\bar{\lambda}$ is an eigen value A^{ϕ} and conversely.